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ABSTRACT

This study involved the extensive testing and interviewing of 102 primary grade pupils, representing a sampling from diverse school environments, to study the capabilities and difficulties evidenced by primary-grade pupils in solving open addition and subtraction sentences. Phase 1 of the study identified the specific incorrect procedures commonly used by pupils in completing such sentences. Since the major incorrect procedures were analyzed as representing difficulties in reading number sentences correctly and with understanding, Phase 2 investigated the importance of (a) reading such sentences and (b) being able to demonstrate their meaning with manipulatives as determiners of ability to solve them. (Author/MN)

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An Analysis of Incorrect Procedures Used by Primary Grade Pupils in Solving Open Addition and Subtraction Sentences

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1978

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An Analysis of Incorrect Procedures Used by Primary Grade Pupils in Solving Open Addition and Subtraction Sentences

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Most present-day elementary school arithmetic curricula devote considerable attention to the solution of open sentences. This is partly a result of the emphases, in modern mathematics, on the pupils acquiring a broad understanding of the general language and structure of mathematics at a fairly early age. It may also be based on the assumption that many applications of math, both in "story problems" found in the curriculum and practical problems encountered in daily life, can be solved most easily if formulated as open sentences.

The emphasis on open sentences in the curriculum has been accompanied by some attention to research on how pupils solve such problems. Some research (Suppes, 1965) (Weaver, 1971) (Grouws, 1972) has attempted to identify the characteristics of the sentences that are associated with differences in difficulty of solution. This work, for example, has been concerned with differences in difficulty between addition and subtraction sentences and between sentences having the operation on the left (e.g., $a + b = c$) and those having the operation on the right (e.g., $c = a + b$). Also investigated has been the effect of the position of the unknown to be found. Other

studies have been concerned with determining the procedures used by students in solving open sentences. Grouws (1974) used an interview technique to identify how students went about solving open addition and subtraction sentences. He identified such steps as (1) direct addition or subtraction, (2) recall, (3) trial substitution with verification and correction as needed, and (4) counting. In another set of studies the researchers have proposed certain counting models, have developed linear equations for predicting the time a student will take to solve a problem using any particular model, and then have determined how well actual time data fit each model. Some such studies (Groen and Parkman, 1972) (Woods, Resnick, and Groen, 1975) have used this procedure and have concluded that a simple counting model best explains the results when students solve simple addition or subtraction problems. However, in another study Parkman (1972) felt that his results indicated that adults answered addition and subtraction facts through retrieval from memory. Groen and Poll (1973) applied this same procedure of analysis of response times, or latencies, to a study of how children solve open addition sentences. Their results indicate that for problems of the form $x + \square = y$ performance is best explained by a model which assumes that the student either counts up from the given addend or counts down from the sum, whichever is quicker.

Of course, an understanding of how pupils solve open sentences correctly is important both for purposes of planning direct instruction and for arranging instructional conditions so as to facilitate pupil invention of their own best solution strategy (Groen and Resnick, 1977). Also of use in planning instruction and in diagnoses and remedial instruction is some knowledge of the incorrect solution strategies used by students. Ginsburg (1976) has

emphasized the importance of this type of information in diagnosing student learning difficulties. Light and Reynolds (1972) have shown how this type of information can be used in improving lesson materials and instructional procedures. Case (1975), using a neo-Piagetian analysis, has pointed to the importance of knowing what incorrect procedure is being used so that the student can be convinced that this procedure does not produce the correct answer. In applying this type of analysis to missing addend problems (one form of open sentence) Case found that a common incorrect procedure used was that of the "addition of givens" (adding whatever two numbers are given in the statement).

The research reported in this paper was an exploratory investigation of the various incorrect solution procedures used by students in attempting to solve a variety of forms of open sentences. It identifies specific incorrect strategies, indicates their relative frequency of occurence, and suggests their relationship to differences among school populations and programs. The study also provides for a follow-up on some of the major incorrect strategies by investigating the extent to which certain capabilities appear to be prerequisites to the correct solution of open sentences and to their application in story problems.

This study was carried out in five elementary schools located in five different communities representing major differences in size and type of community. In each school a sample of approximately 20 primary grade pupils was interviewed and tested through a rather intensive procedure that provided measures of capability in solving each of a number of types of open sentences, of ability to read such sentences correctly, of ability to demonstrate the meaning of each sentence through the use of manipulative materials,

and ability to solve story problems that involved applications of the various sentence types.

Types of incorrect procedures were determined for each type of open sentence using five different measures of ability. These latter five measures are as follows.

Written Test. Pupil was given a paper-and-pencil test and asked to fill in the missing number in the sentence.

Read Orally. Pupil was shown an open sentence and asked to read it aloud. Pupil was expected to read it in the correct sequence and to indicate the place of the unknown by using a word such as "something" or "box" or to fill in the solution number.

Demonstrate. Pupil was shown a sentence and directed to "Show me what this sentence means using those blocks."

Story Problem. Pupil was read a story problem that involved the application of a particular type of open sentence and was expected to provide the answer.

Demonstrate Story Problem. Pupil was asked "Can you explain how you would solve this problem if you pretend that these blocks (or chips) are ...?"

Tables 1 through 7 summarize the procedures used by pupils under each of these testing conditions with each of seven different types of open sentences.

Tables 1 and 2 are not particularly exciting, showing basically that most students have little difficulty with this type of sentence except in

its application to story problems. Here there is some tendency to add the two numbers given or to merely respond by reporting one of these given numbers as the needed answer.

The results summarized in Tables 3 and 4 show, first, that subtraction sentences in which the missing term is in the first position are much more difficult than those in which the missing term is in the second position. This agrees with the finding from many past studies (Suppes, 1965; Weaver, 1971; Grouws, 1972). However, an examination of the types of errors made, as summarized in Table 4, may offer some suggestion concerning the reason for this difference. This shows that with simple written problems of the form $\square - b = c$ the most common incorrect procedure followed is that of "subtracting givens", that is looking at the two numbers given and subtracting the smaller from the larger. This suggests that for many students a number sentence is not an expression that states a relationship and that must be read in the sequential order given. To these students a simple number sentence is only a listing of two numbers and an operation sign, and the operation is to be applied to these numbers in the possible, or most convenvient, manner. Note that with the type of sentence that forms the basis for the Table 3 data the application of the "subtract givens" rule will result in the correct answer. This suggests that one important reason why the subtraction sentences with the missing term in the second position are the easier is that both those persons who apply the correct procedure and those who apply the most common incorrect rule ("subtract givens") will obtain the correct answer.

A study of the data in Table 4 on ability to read indicates that the majority of students could verbalize this sentence correctly (e.g., with

the sentence □ - 4 = 3 they would say "Something minus 4 equals 3", or some equivalent expression, or would go directly to the correct substitution and say "7 minus 4 equals 3"). However, this ability to read the problem (in the above sense of verbalizing) does not necessarily result in ability to solve the problem, as can be seen.

A study of Table 4 data on demonstrating "what the sentence means" by using manipulatives also suggests the limited capability represented by a student's ability to verbalize a number sentence. Here, it is obvious that many persons who can verbalize the sentence do not comprehend it to the extent of being able to show what it means. In this type of demonstration many students represented the subtraction of givens (e.g., with the sentence □ - 4 = 3 they would demonstrate that 4 minus 3 equals 1). Another substantial number solved the problem backward (i.e., with □ - 4 = 3, they would show that 4 minus 1 equals 3).

The data on procedures used in solving story problems as reported in Tables 3 and 4 shows some variety in incorrect procedures, including giving a number from the problem, adding or subtracting the numbers given, and using the backward operation.

A further examination of possible prerequisite relationships between ability to read a sentence correctly, ability to demonstrate its meaning, and ability to solve it correctly that are suggested by the foregoing data is presented in a second phase of data analysis to be presented below.

Addition and Subtraction Sentences with Operation on Right

The everyday experience of most primary-grade teachers as well as results from research (Weaver, 1973) indicate that pupils have much more

difficulty in solving open sentences in which the operation notation is on the right of the equal sign (e.g., 6 = □ + 2) than in solving sentences with the operation on the left (e.g., □ + 2 = 6).

Tables 5 and 6 summarize results obtained from pupils' work with addition sentences of this general form. It can be seen that there are more errors here than are shown in Tables 1 and 2, examined previously, based on addition sentences having the more standard form of showing the operation on the left. Tables 5 and 6 show that the most common error in solving written sentences of this type is that of "adding givens." It can be seen that this error is somewhat more prevalent here than it is with the operation-on-left sentences (Tables 1 and 2). In reading the operation-on-right sentences almost 50% of the students read these sentences "backward", or by starting on the right. That is, they would read the sentence 6 = □ + 2 by saying "2 plus something equals 6." Of course, because of the commutative property of addition, this reading is not exactly incorrect and expresses a sentence that has the same missing term as does that sentence read from left-to-right. However, because of possible insights that might be provided by the specific reading procedure followed, this type of performance is placed in a separate category here. It can be seen that the most common error in demonstrating such sentences is that of adding the givens.

Table 7 summarizes results from measuring pupil capabilities with respect to subtraction sentences with the operation on the right. In solving written sentences of this type the common error was that of reading the sentence from right to left. That students did this quite commonly is evidenced both by the answer they gave to the written sentences and the way they actually

verbalized such a sentence when asked to read it. Note that the proportion of students who read (verbalize) from right to left with the subtraction sentences is approximately the same and even a little less than the proportion doing this with the addition sentences. However, since subtraction is not commutative, students who read the sentence $4 = 7 - \square$ as "Something minus 7 equals 4" are reading it incorrectly and will obtain an incorrect solution if they use this as a basis for solving the given sentence.

Summary of Incorrect Methods Used in Solving Open Sentences

The data presented in the foregoing section of this report may be summarized with respect to each type of open sentence.

Addition Sentences, Missing Addend, Operation Left ($a + \square = c$, $\square + b = c$)

Sentences of this type were difficult for very few children in this sample. The most common incorrect procedure used was to "add givens."

Subtraction Sentences, Second Term Missing, Operation Left ($a - \square = c$)

No major incorrect procedure was found here. A "subtraction of givens" rule, if followed would yield correct answer.

Subtraction Sentences, First Term Missing, Operation Left ($\square - b = c$)

Here the "subtraction of givens" was the common error in solving the written sentences. Both "subtraction of givens" and "solving backward" were common errors in reading a sentence and in demonstrating its meaning. With story problems, reporting one of the numbers given in the story as the answer was the most common error.

Addition Sentences, Missing Addend, Operation Right ($c = a + \square$, $c = \square + b$)

The "addition of givens" was the common error in solving written sentences and in demonstrating the meaning of a sentence. In reading the sentence, a common practice was to read it from right to left.

Subtraction Sentences, Third Term Missing, Operation Right ($c = a - \square$)

Again, reading from right to left was the common error in solving written sentences, in reading, and in demonstrating "what the sentence means" with manipulatives.

The foregoing survey of the incorrect solution procedures employed by students in solving open sentences indicates that the type of incorrect procedures used depends upon the exact form of the sentence involved. However, most of the inadequate strategies employed are based on an incorrect reading of the sentence and an inadequate comprehension of what it really means. This latter generalization suggests the hypotheses that (1) ability to solve a number sentence will depend upon the ability to read (verbalize) it correctly and (2) ability to solve a number sentence will depend upon the ability to explain what the sentence means. Although the data obtained in this study do not permit a rigorous testing of these hypotheses, they do permit some exploratory investigation of their plausibility.

A re-examination of Tables 1, 2, and 3 will show that almost all pupils were able to read correctly the open sentences of the types involved ($a + \square = c$, $\square + b = c$, and $a - \square = c$). As a result, these data did not permit an examination of the relationship of reading ability to the ability to solve the sentence. However, student performance on sentences of the type $\square - b = c$ provides some basis for an analysis of this relationship.

Table 8 provides data in the form of contingency tables showing the relationship between ability to read this type of sentence correctly and ability to solve for the unknown term. These tables seem to suggest that reading the sentence is probably an essential pre-requisite to being able to solve it. By looking at the data in the top row of each of the three tables (the row showing those persons who were able to solve open sentence) one can note the reading performance of those who could solve the sentence. With two exceptions, all persons who were able to solve the problem were able to read that type of sentence correctly. On the other hand, the fact that many students who were able to read the sentence were still not able to solve it suggests that being able to read (verbalize) the type of sentence, while being a necessary pre-requisite, is not a "sufficient" pre-requisite. Additional abilities are needed.

Table 9 provides data that indicate that the relationship between ability to read the number sentence and to solve story problems based on that type of sentence is not as obvious as that found in Table 8. For two of the schools this ability to read the number sentence appears to be an essential prerequisite to being able to solve story problems based on that type of sentence. However, in the other three schools this relationship does not exist. Overall, these data do not appear to provide the basis for any type of general conclusion concerning this relationship.

The second hypothesis suggested above was that ability to solve a number sentence correctly will depend upon the ability to explain what that sentence means. The method used to measure this type of comprehension in the present study was to present the student with a written example of this

type of number sentence and ask "Can you show me what this sentence means by using these blocks?" This is the "demonstrate" measure. Table 10 shows the relationships between this measure and ability to solve this type of sentence. Obviously these data fail to support the hypothesis. A substantial number of students who failed to demonstrate this type of sentence correctly could still arrive at a correct solution. In speculating on reasons for this lack of relationship two possibilities must be considered. One is that some pupils can solve these sentences without understanding them the other, that this "demonstrate" test is not the appropriate measure of understanding. The first explanation would be plausible, for example, if some students were getting their answers by copying from another student. Since the present investigation involved individual testing, copying was impossible. Another possibility would be that students responded on the basis of memory. However, the application of memorized "subtraction facts" to this type of open sentence would also seem to require some type of understanding. Because of the lack of a readily plausible explanation as to how pupils could solve an open sentence of this degree of complexity without some type of "understanding" of it and because accepting the proposition that some students respond correctly without understanding the sentence is not conducive to further exploration of what is required for solution, the present investigators choose to assume that the "demonstrate" test used here is not an adequate measure of pupil comprehension of the meaning of an open sentence.

For a further exploration of the nature of the "reading" and "demonstrate" measures used in the foregoing analysis it is useful to look at the relationship between these two measures. Table 11 provides data on this. These

data indicate that ability to read is a necessary pre-requisite to being able to demonstrate. All persons who were able to demonstrate the meaning of this type of sentence were able to read it correctly. Again, however, many students who could read the sentence could not demonstrate its meaning. As was true both with solving this type of sentence and with solving story problems based on this type of sentence, being able to read the sentence correctly was a necessary pre-requisite but not a sufficient pre-requisite to being able to demonstrate its meaning.

Discussion

The limited data presented in the foregoing analysis have served to suggest that ability to read a sentence is an important determiner of ability to solve it and of ability to demonstrate its meaning with manipulatives. However, the data do not support the hypothesis that ability to demonstrate the meaning of a number sentence through the use of manipulatives is a pre-requisite to solving it. At first consideration the finding with respect to the importance of reading may appear merely to be support for the importance of an obvious prerequisite. If students are to solve a sentence, they must be able to read it. However, both the data and the subjective impressions obtained from the inverviews in this study suggest that the role of careful and correct reading may not be given sufficient emphasis in teaching. It was noted, for example, that many pupils who read an incomplete sentence correctly at first show no reluctance to re-read it in a different order if this results in a sentence that can be solved more easily. Also, in checking the correctness of an answer, some students who originally read the sentence correctly will read it in an incorrect manner if this supports their solution procedure.

Some further insights concerning the relationship of the "reading" measure to the "demonstrate" measure can be obtained from the notes that are a part of the record of this interview testing. The typical errors on the "demonstrate" measure involved the student demonstrating an incorrect version of the sentence. In many cases the student would read the sentence correctly, would solve it incorrectly, and then demonstrate the incorrect solution. That is, where one might anticipate that students who arrive at an incorrect solution to a sentence would discover the error when they demonstrate the sentence, what happens instead is that they demonstrate the procedure they followed in obtaining the incorrect answer. This might be interpreted as indicating that students do not see a number sentence as something whose meaning is dependent upon the structure and order of the components of the sentence. They see the sentence as consisting only of some symbols that can be reordered if this is necessary to make it solvable.

These inconsistencies that pupils display in reading number sentences may derive, to some degree, from the fact that a simple addition or subtraction sentence can be read in two different ways. In one form it can be read as describing an operation, e.g.,"When 2 is added to 3 the sum is 5." In another form this same sentence would be interpreted as saying "The number named by 2 + 3 is the same number as that named by the numeral 5." It may be that the difficulty most students have with open sentences is due to the fact that they tend to read such sentences only as descriptions of an operation. For example, the fact that so many students read a sentence backward (see, for example, the data in Table 7 showing performance on the sentence $c = a - \square$), particularly when the operation is on the right,

would appear to indicate that they first look for the description of the operation and "know" that this should come first and be followed be the "answer." It would seem that if students interpreted a sentence as giving two names for the same number they would feel much less need for reading the operation first and would be more likely to read the sentence in the correct order.

These rather broad inferences from the results of this study would seem to suggest further investigations such as an experimental study which would examine the effect of instruction in careful reading of number sentences on ability to solve them.

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Table 1. Numbers of Students In Each School Giving the Correct Response and Number Using Various Specific Incorrect Procedures In Responding to Incomplete Number Sentences Under Four Testing Conditions (Sentence Form: a + □ = c)

Response and Procedures	Give Answer Written Test					Read Sentence Orally					Demonstrate With Blocks					Solve Story Problems Involving This Form				
	Sch A	Sch B	Sch C	Sch D	Sch E	Sch A	Sch B	Sch C	Sch D	Sch E	Sch A	Sch B	Sch C	Sch D	Sch E	Sch A	Sch B	Sch C	Sch D	Sch E
Correct	16	16	21	14	16	18	19	22	18	19	19	14	22	18	18	16	7	16	15	15
Incorrect																				
Add givens	1	4	1	1	1	2	1		1		1	4		1	1	2	8	4	1	1
Subtract givens																				
Give number a																		1		
Give number c				1								1				2	2	1	1	2
Comput. error	2			1	1							1							2	
Omit	1				1												1			
Unexplainable	1			2		1					1					1	2			1
Total	21	20	22	19	19	21	20	22	19	19	21	20	22	19	19	21	20	22	19	19

Table 2. Numbers of Students In Each School Giving the Correct Response and Number Using Various Specific Incorrect Procedures In Responding to Incomplete Number Sentence Under Four Testing Conditions (Sentence Form: ☐ + b = c)

Response and Procedure	Give Answer Written Test					Read Sentence Orally					Demonstrate with Blocks					Solve Story Prob. Involving This Form				
	Sch A	Sch B	Sch C	Sch D	Sch E	Sch A	Sch B	Sch C	Sch D	Sch E	Sch A	Sch B	Sch C	Sch D	Sch E	Sch A	Sch B	Sch C	Sch D	Sch E
Correct	14	17	21	17	17	18	18	22	19	19	18	13	21	17	18	17	5	15	13	17
Incorrect																				
Add givens	1	3	1		1	1	1				3	5	1	2	1	3	8	3	2	1
Subtract givens																				
Give number b																1	2			1
Give number c	1																3	1		
Comput. error	1																	1	4	
Omit	3			2	1							1						1		
Unexplainable	1					2	1					1					2	1		
Total	21	20	22	19	19	21	20	22	19	19	21	20	22	19	19	21	20	22	19	19

Table 3. Numbers of Students In Each School Giving the Correct Response and Number Using Various Specific Incorrect Procedures in Responding to Incomplete Number Sentences Under Four Testing Conditions (Sentence Form: a - □ = c)

Response and Procedure	Give Answer Written Test					Read Sentence Orally					Demonstrate with Blocks					Solve Story Prob. Involving This Form				
	Sch A	Sch B	Sch C	Sch D	Sch E	Sch A	Sch B	Sch C	Sch D	Sch E	Sch A	Sch B	Sch C	Sch D	Sch E	Sch A	Sch B	Sch C	Sch D	Sch E
Correct	14	16	18	13	16	19	18	22	18	19	19	19	22	19	18	15	12	19	17	16
Incorrect																				
Add givens				1							1	1					1	1		
Subtract givens						2	1		1											
Give number a															1		3			
Give number c																2	4	1		2
Comput. error	4	3	1	2	2											3			2	1
Omit	3	1	1	2	1				1							1				
Unexplainable			2	1			1											1		
Total	21	20	22	19	19	21	20	22	19	19	21	20	22	19	19	21	20	22	19	19

Table 4. Number of Students In Each School Giving the Correct Response and Number Using Various Specific Incorrect Procedures In Responding to Incomplete Number Sentences Under Four Testing Conditions (Sentence Form: □ - b = c)

Response and Procedure	Give Answer Written Test					Read Sentence Orally					Demonstrate with Blocks					Solve Story Prob. Involving This Form				
	Sch A	Sch B	Sch C	Sch D	Sch E	Sch A	Sch B	Sch C	Sch D	Sch E	Sch A	Sch B	Sch C	Sch D	Sch E	Sch A	Sch B	Sch C	Sch D	Sch E
Correct	4	6	11	3	2	17	15	20	12	16	5	7	12	8	4	12	7	10	17	11
Incorrect																				
Add givens												1					2			
Subtract givens	13	6	10	12	12	2	1	1			7	3	2	3	3		4	3		1
Give number b																8	3	7		5
Give number c																	2			
Comput. error	1	1	1	1	1											1				
Omit	3	6		2	4					1	2	2		1	2				1	
Unexplainable (b - □ c)		1		1		1					1				1		1	1	1	2
Solve backward						1	4	1	7	2	6	7	8	7	9		1	1		
Total	21	20	22	19	19	21	20	22	19	19	21	20	22	19	19	21	20	22	19	19

Table 5. Number of Students In Each School Giving the Correct Response and Number Using Various Specific Incorrect Procedures In Responding to Incomplete Number Sentence Under Three Testing Conditions (Sentence Form: c = a + □)

Response and Procedure	Give Answer Written Test					Read Sentence Orally					Demonstrate Meaning of Sentence With Blocks				
	Sch A	Sch B	Sch C	Sch D	Sch E	Sch A	Sch B	Sch C	Sch D	Sch E	Sch A	Sch B	Sch C	Sch D	Sch E
Correct	12	2	13	9	14	10	6	16	10	12	15	14	20	14	17
Incorrect															
Add givens	4	8	6	6	2	1	1	1	2		4	4	2	4	1
Subtract givens						1					1				
Comput. error	1		1	3	1										
Omit	4			1	1						1				
Unexplainable			2		1							2		1	1
Read Right to left*						9	13	5	7	7					
Total	21	20	22	19	19	21	20	22	19	19	21	20	22	19	19

*Not incorrect with this sentence form. Noted here to permit comparison with pupil performance on subtraction sentences of this general from. See Table 7.

Table 6. Number of Students In Each School Giving the Correct Response and Number Using Various Specific Incorrect Procedures In Responding to Incomplete Number Sentences Under Three Testing Conditions (Sentence Form c = □ + b)

Response and Procedure	Give Answer Written Test					Read Sentence Orally					Demonstrate Meaning of Sentence With Blocks				
	Sch A	Sch B	Sch C	Sch D	Sch E	Sch A	Sch B	Sch C	Sch D	Sch E	Sch A	Sch B	Sch C	Sch D	Sch E
Correct	13	12	20	15	15	9	4	15	5	11	16	12	18	15	14
Incorrect															
Add givens	1	6	2	2	1	1	1		1	1	4	4	3	3	3
Subtract Givens															1
Comput. error	2												1		
Omit	5	1		1	2						1	3			
Unexplainable		1		1	1		1					1		1	1
Read Right to left*						11	14	7	13	7					
Total	21	20	22	19	19	21	20	22	19	19	21	20	22	19	19

*Not incorrect with this sentence form. Noted here to permit comparison with pupil performance on subtraction of this form. See Table 7.

Table 7. Number of Students In Each School Giving the Correct Response and Number Using Various Specific Incorrect Procedures In Responding to Incomplete Number Sentences Under Three Testing Conditions (Sentence Form c = a - □)

Response and Procedure	Give Answer Written Test					Read Sentence Orally					Demonstrate Meaning of Sentence With Blocks				
	Sch A	Sch B	Sch C	Sch D	Sch E	Sch A	Sch B	Sch C	Sch D	Sch E	Sch A	Sch B	Sch C	Sch D	Sch E
Correct	5	7	15	8	11	11	8	17	9	11	8	7	19	12	10
Incorrect															
Add givens											2	1	1	1	
Subtract givens						2	1		1		1	1	1		2
Give number a		1												1	
Give number c											1				
Comput. error	1	1	1	2	1										
Omit	5	2		1	3						4	2	1		
Unexplainable	3	2	2	2				1			1	1		1	2
Read right to left	7	7	4	6	4	8	11	4	9	8	4	8		4	5
Total	21	20	22	19	19	21	20	22	19	19	21	20	22	19	19

Table 8. Contingency Tables Showing Relationship of Performance on Reading An Open Sentence To Ability to Solve That Type of Sentence (Sentence Type $\square - b = c$)

School A

Solve Sent.	Read Sent. F	Read Sent. P
P	0	11
F	2	9

School B

Solve Sent.	Read Sent. F	Read Sent. P
P	0	4
F	4	13

School C

Solve Sent.	Read Sent. F	Read Sent. P
P	1	5
F	4	10

School D

Solve Sent.	Read Sent. F	Read Sent. P
P	1	2
F	6	10

School E

Solve Sent.	Read Sent. F	Read Sent. P
P	0	2
F	3	14

Table 9. Contingency Tables Showing Relationship of Performance on Reading An Open Sentence to Ability to Solve Story Problems Based on That Type of Sentence (Sentence Type □ -b=c)

School A

Solve Story \ Read Sent.	F	P
P	0	10
F	2	10

School B

Solve Story \ Read Sent.	F	P
P	0	12
F	4	5

School C

Solve Story \ Read Sent.	F	P
P	3	4
F	2	11

School D

Solve Story \ Read Sent.	F	P
P	6	11
F	1	1

School E

Solve Story \ Read Sent.	F	P
P	2	9
F	1	7

Table 10. Contingency Tables Showing Relationship of Ability to Demonstrate the Meaning of a Given Type of Open Sentence to Ability to Solve that Type of Sentence (Sentence Type $\square - b = c$)

School A

Solve Sent. \ Demonstrate	F	P
P	4	7
F	6	5

School B

Solve Sent. \ Demonstrate	F	P
P	2	2
F	14	3

School C

Solve Sent. \ Demonstrate	F	P
P	2	4
F	11	3

School D

Solve Sent. \ Demonstrate	F	P
P	2	1
F	9	7

School E

Solve Sent \ Demonstrate	F	P
P	0	2
F	15	2

Table 11. Contengency Tables Showing Relationship of Ability to Read an Open Sentence Type Correctly and Ability to Demonstrate the Meaning of that Type of Sentence by Using Manipulatives (Sentence Type □ - b = c)

School A

Demonstrate \ Read Sent.	F	P
P	0	12
F	2	8

School B

Demonstrate \ Read Sent.	F	P
P	0	5
F	4	12

School C

Demonstrate \ Read Sent.	F	P
P	0	7
F	5	8

School D

Demonstrate \ Read Sent.	F	P
P	0	8
F	7	4

School E

Demonstrate \ Read Sent.	F	P
P	0	4
F	3	12